To "B" or Not To "B": A Welfare Analysis of Breaking Up Monopolies in an Endogenous Growth Model

Danyang Xie*

Hong Kong University of Science and Technology

April, 2000

Abstract

We study the welfare consequences of a government regulation that forces a patented equipment to be supplied by n independent producers. On the one hand, such a regulation hurts the value of a patent and therefore reduces activities in the R&D sector. On the other hand, the enhanced competition for the equipment improves efficiency in the manufacturing sector. Should we break up monopolies protected by intellectual property rights? The answer is no in a Romer-type growth model but we have sufficient reason to believe that the answer could be yes in a model advocated by Jones (1995).

JEL Classification Numbers: O31, O38, O41

Keywords: R&D, Growth, Competition Policy

*I thank Professor Kazuo Nishimura for organizing a Kyoto workshop on growth and indeterminacy, during which the idea of this paper was conceived. I thank Sergio Rebelo, Guofu Tan, Eden Yu and seminar participants at CUHK for helpful comments. Financial support from a HKUST DAG grant and a grant from Research Grants Council of the HKSAR (Project No. HKUST6003/99H) is gratefully acknowledged.

1 Introduction

To "B" or not to "B". Am I suggesting that we should ask Shakespeare for economic insights? Hardly. The "B" in the title stands for "Break up". "Breaking up" monopolies protected by a patent involves a clear trade-off. On the one hand, a "break up" enhances competition and reduces deadweight loss. On the other hand, a "break up" reduces the value of a patent and hence adversely affects the incentive of conducting R&D. This trade-off is essentially the conflict between the patent law and the antitrust law, a conflict that is difficult to resolve without a formal model. "To Break up or Not To Break up", that indeed is the question.

Whereas the currently heated debate on the U.S. government's case against Microsoft focuses on the anticompetitive act of Microsoft to extend its legitimate monopoly of operating system to other product markets, I ask whether there should be a government regulation that forces any patented product to be supplied by n independent producers. I intend to conduct a welfare analysis of such a regulation in an endogenous growth model.

The main results are obtained with a R&D-based Romer model (1990). Clearly, if we are only concerned with the long-term rate of growth, a "break up" is never an attractive option: a "break up" hurts the value of a patent and therefore reduces the activities in the R&D sector, leading to a lower long-term growth rate. What we should be concerned with, however, is the welfare. A "break up" enhances competition among the suppliers of specialized equipment so that the manufacturing sector will benefit from a lower input price.

What are the relevant factors for a trade-off? Will a "break up" ever have a chance to raise the welfare of a representative agent?

Endogenous growth models have been widely used to evaluate government fiscal policies (see, for example, Barro 1990, Rebelo 1991, Barro and Sala-i-Martin 1992, Glomm and Ravikumar 1994, Stokey and Rebelo 1994 and Mino 1996). Most of these analyses focus on the long-term rate of growth. Welfare analysis is usually limited to models of AK type. Devarajan, Xie and Zou (1998) investigate the welfare implications of government intervention when a lump-sum tax is not available. To obtain analytical results, they use logarithmic utility functions, the Cobb-Douglas production function and the 100 percent rate of capital depreciation.

I think that endogenous growth models are also the natural vehicles to study government policies besides those dealing with fiscal issues. A number of papers examine the issue of optimal patent length (see, for example, Davidson and Segerstrom 1992, Horowitz and Lai 1996, and Futagami, Mino and Ohkusa 1999). This paper represents a first attempt at examining a government regulation that breaks up monopolies of patented equipment. I find that in Romer (1990), a “break up” can be very costly if the long-lived representative agent’s intertemporal rate of substitution is high. In a numerical example, a regulation that breaks up a monopoly into two independent firms lowers welfare by 63 percent.

The literature on the equity premium, however, indicates that a low intertemporal rate of substitution receives more empirical support. When the representative agent’s intertemporal rate of substitution is low, a “break up” of a monopoly into two independent suppliers reduces the welfare by only 0.3 percent. Hence, “To Break up or Not To Break up”, is essentially a draw.

More importantly, if the economy works as in a model advocated by Jones (1995a), a “break up” may actually be the better choice. In Jones (1995a), a “break up” affects only the short-term growth rate of R&D but it has no long-term growth effect. This considerably lowers the welfare cost of a “break up”. I speculate that in this model, an optimal $n > 1$ exists.

The computing technique I use in this paper is called “reverse shooting”

(Judd 1998, pp. 355-57). In a Romer-type model, "reverse shooting" works well because the equilibrium path can be described by a system of three differential equations with *one* initial condition and two terminal conditions. In a Jones-type model, "reverse shooting" is problematic because the equilibrium path is described by a system of four differential equations with *two* initial conditions and two terminal conditions. Although Judd (1998) addresses briefly the issue of multidimensional reverse shooting (pp. 360-61), the computation is in fact much more difficult than it appears[1].

The rest of the paper is organized as follows. In Section 2, I present an extended version of the Romer model, allowing for imperfect substitutability among different types of equipment. Intuitively, if different types of equipment are highly substitutable, the welfare cost of a "break up" would be insignificant. This is confirmed in our comparative dynamics analysis in Section 3. Section 4 concludes.

2 An Extended Romer Model

The model is an extended version of Romer (1990). I deviate from his original work in two aspects. First, in the manufacturing sector, the production function is modified so that different types of equipment are no longer independent; they are assumed to be imperfect substitutes. Second, the government makes a regulation that requires each type of equipment to be supplied by n independent producers.

[1]Detailed differential equations for the Jones model is available in the appendix if anyone has an interest in solving them.

2.1 The Manufacturing Sector

The production function in the manufacturing sector is given by

$$Y = H_Y^{\alpha} L^{\beta} \left(\int_0^A x_a^{\gamma/\xi} da \right)^{\xi}, \tag{1}$$

where H_Y is the amount of human capital (managers and engineers) employed in the manufacturing sector, L is the unskilled labor, which is in a fixed supply, x_a is the amount of type a equipment and A is the number of different types of equipment. I assume that $\alpha + \beta + \gamma = 1$ so that the production function exhibits constant returns to scale in $(H_Y, L, \{x_a\}_0^A)$. ξ is in interval $(\gamma, 1)$ so that different types of equipment are imperfect substitutes. When $\xi = \gamma$, the production function is neoclassical in the sense that all equipment are treated as perfect substitutes. When $\xi = 1$, it is reduced to Romer model.

A similar production function is used in Benhabib, Perli and Xie (1994). There, the attention was on the case when $\xi > \gamma$. They show that when different types of equipment display high enough complementarity, the modified Romer model exhibits indeterminacy.

2.2 The R&D Sector

The production of new designs in the R&D sector is given by

$$\dot{A} = \delta H_A A, \tag{2}$$

where H_A is the amount of human capital employed in the R&D sector and δ is a productivity parameter.

2.3 The Preferences

I assume that there is a long-lived representative individual with preferences as shown by

$$\max \int_0^\infty \left[\frac{C^{1-\sigma} - 1}{1-\sigma} \right] e^{-\rho t} \; dt, \tag{3}$$

where C is the consumption of the single final good, σ is the inverse of the intertemporal rate of substitution and ρ is the rate of time preference.

With this CES utility function, we know that the optimality consumption path satisfies the following condition:

$$\frac{\dot{C}}{C} = \frac{r-\rho}{\sigma}, \tag{4}$$

where r is the real interest rate and could be time varying.

2.4 The Market Structure

The government makes a regulation that requires each type of equipment to be supplied by n independent producers. I assume that these n producers play a Cournot–Nash oligopoly game. An increase in n certainly lowers the value of a patent. Kremer (1998) has advocated a patent buyout by government, which eliminates the monopoly price distortions while increasing incentives for original research. The implementation of a buyout however is problematic because the price of a patent is subject to collusion. Furthermore, large-scale patent buyouts would be unthinkable if the government has to use distortionary taxes to finance these purchases. In my paper, the value of a patent is market driven and the only decision that the government makes is to set n, the number of licenses. There are an infinite number of potential producers, each of which (including the inventor if he wishes) will submit a bid for one of the n licenses to become the supplier. The highest n bidders win the licenses and the value of the patent is given by the sum of the n winning bids.

To see the exact relationship between the number of licenses and the value of the patent, note that the demand curve for type a equipment is given by

$$p_a = \gamma H_Y^\alpha L^\beta \left(\int_0^A x_a^{\gamma/\xi} da \right)^{\xi - 1} x_a^{\gamma/\xi - 1}, \tag{5}$$

where p_a is the rental price of type a equipment.

Given the demand curve (5), supplier i of type a equipment, taking the quantity supplied by its $n-1$competitors $({}_{-i}x_a)$ as given, chooses its optimal quantity (x_{ai}) by solving the following problem:

$$\max \ \pi_{ai} = p_a(x_{ai}, {}_{-i}\, x_a) x_{ai} - r x_{ai}$$

where I follow Romer (1990) in assuming that one unit of final goods can be converted into one unit of type a equipment as long as the producer holds the patent. To see that the marginal rental cost of the producer is the real interest rate r, note that a piece of equipment is assumed to be perfectly durable. In this case, if we use letter m to denote the marginal rental cost of a supplier, the one-to-one conversion between a unit of final consumption goods and the equipment requires

$$1 = \int_t^\infty m(\tau) e^{-\int_t^\tau r(s) ds} d\tau$$

for any time t. Differentiating the above identity with respect to t, we obtain that $m(t) = r(t)$.

The profit maximization problem yields a first-order condition

$$r = (\gamma/\xi - 1) \frac{p_a}{x_a} x_{ai} + p_a. \tag{6}$$

Note that all these suppliers are identical, namely at equilibrium,

$$x_{ai} = \frac{1}{n} x_a \text{ for all } i.$$

Hence, the equilibrium rental price of the Cournot-Nash oligopoly game can be obtained from (6):

$$p_a = \frac{r}{1-(1-\gamma/\xi)/n} \tag{7}$$

which reduces to $p_a = r/\gamma$ if $\xi = 1$ and $n = 1$, the same as in Romer (1990).

The total profit that can be made by these suppliers of type a equipment is

$$\begin{aligned} \pi_a &= \Sigma_{i=1}^{n} \pi_{ai} \\ &= \left[\frac{(1-\gamma/\xi)/n}{1-(1-\gamma/\xi)/n}\right] r x_a. \end{aligned}$$

Thus the value of a patent for a new design, a, invented at time t is equal to the present value of the entire profit stream in the future:

$$P_a(t) = \int_t^{\infty} \pi_a(\tau) e^{-\int_t^{\tau} r(s)ds} d\tau.$$

Because of the symmetry in all the new equipment invented at time t, I can replace the lower-case letter a by the upper-case letter A in the equations above; and I sometimes omit the subscript all together when it will not cause any confusion. In particular, we have

$$\begin{aligned} P_A(t) &= \int_t^{\infty} \pi(\tau) e^{-\int_t^{\tau} r(s)ds} d\tau \\ &= \int_t^{\infty} \left[\frac{(1-\gamma/\xi)/n}{1-(1-\gamma/\xi)/n}\right] r x \, e^{-\int_t^{\tau} r(s)ds} d\tau, \end{aligned} \tag{8}$$

where x can be read from equations (5) and (7):

$$\frac{r}{1-(1-\gamma/\xi)/n} = \gamma H_Y^{\alpha} L^{\beta} A^{\xi-1} x^{\gamma-1}. \tag{9}$$

2.5 Human Capital Allocation

I assume that total stock of human capital is fixed. It is denoted by H. In equilibrium, the allocation of human capital is such that the marginal revenue product of human capital is equalized in the two sectors. That is,

$$\alpha H_Y^{\alpha-1} L^\beta A^\xi x^\gamma = P_A \delta A, \tag{10}$$

where $H_Y + H_A = H$.

2.6 Transitional Dynamics

Understanding transitional dynamics is essential in our evaluation of "To Break up or Not To Break up". If we are only concerned with long-term growth, "Not To Break up" ($n = 1$) is clearly the best choice. From the welfare point of view, however, it is questionable whether $n = 1$ continues to be optimal because an increase in n benefits the manufacturing sector.

The question therefore is this: Can the benefit to the manufacturing sector be large enough to compensate for the reduced rate of growth in R&D?

To calculate the welfare, I need to study the transitional dynamics. The traditional method of linearization around the steady state does not work well here because, when n changes, the corresponding steady state also changes. Our starting point can only be close to one but not all the steady states. The errors from linear approximation may contaminate the comparisons among the welfares attainable under different n.

The computing technique that I use is called "reverse shooting", described in Judd (1998). This method works well when the system of differential equations has only *one* initial condition.

Following Mulligan and Sala-i-Martin (1991), I define state-like and control-like variables as follows:

$$z = A^{\xi-\gamma}K^{\gamma-1} \qquad \text{a state-like variable,} \tag{11}$$

$$q = C/K \qquad \text{a control-like variable,} \tag{12}$$

and H_Y is a control variable.

Once we obtain the paths of these three variables, we can easily solve for other variables, such as, $A(t),\ K(t),\ x(t),\ C(t)$ and $r(t)$.

To obtain the differential equations governing the three variables, note that from equation (8),

$$\begin{aligned}
\frac{\dot{P}_A}{P_A} &= r - \frac{\pi}{P_A} \\
&= r - \frac{(1-\gamma/\xi)\,\gamma}{n\alpha}\delta H_Y \quad \text{use equation (9)} \\
&= (1-(1-\gamma/\xi)/n)\,\gamma H_Y^{\alpha}L^{\beta}z - \frac{(1-\gamma/\xi)\,\gamma}{n\alpha}\delta H_Y.
\end{aligned}$$

Rewrite equation (10) as

$$\alpha H_Y^{\alpha-1}L^{\beta}A^{\xi-\gamma-1}K^{\gamma} = P_A\delta. \tag{13}$$

Computing growth rates from both sides of the above equation, we obtain

$$(\alpha-1)\frac{\dot{H}_Y}{H_Y} + (\xi-\gamma-1)\,\frac{\dot{A}}{A} + \gamma\frac{\dot{K}}{K} = \frac{\dot{P}_A}{P_A}.$$

Hence,

$$\begin{aligned}
\frac{\dot{H}_Y}{H_Y} &= \frac{1}{1-\alpha}\left[(\xi-\gamma-1)\,\frac{\dot{A}}{A} + \gamma\frac{\dot{K}}{K} - \frac{\dot{P}_A}{P_A}\right] \\
&= \frac{1}{1-\alpha}\left[(\xi-\gamma-1)\,\delta(H-H_Y) - \gamma q + \frac{(1-\gamma/\xi)\gamma}{n}H_Y^{\alpha}L^{\beta}z + \frac{(1-\gamma/\xi)\,\gamma}{n\alpha}\delta H_Y\right].
\end{aligned} \tag{14}$$

Obtaining the dynamic equations for z and q is straightforward:

$$\begin{aligned}\frac{\dot{z}}{z} &= (\xi-\gamma)\frac{\dot{A}}{A}+(\gamma-1)\frac{\dot{K}}{K} \\ &= (\xi-\gamma)\delta(H-H_Y)+(\gamma-1)\left(H_Y^{\alpha}L^{\beta}z-q\right) \qquad (15)\end{aligned}$$

and

$$\begin{aligned}\frac{\dot{q}}{q} &= \frac{\dot{C}}{C}-\frac{\dot{K}}{K} \\ &= \frac{(1-(1-\gamma/\xi)/n)\,\gamma H_Y^{\alpha}L^{\beta}z-\rho}{\sigma}-\left(H_Y^{\alpha}L^{\beta}z-q\right). \qquad (16)\end{aligned}$$

To use the "reverse shooting" method, I need to know the steady-state values of q, z, and H_Y, which are denoted q^*, z^* and H_Y^*. If $z(0)=z^*$, the economy is on a balanced growth path.

2.7 Balanced Growth Path

To find the balanced growth path, set $\dot{H}_Y$, $\dot{z}$ and $\dot{q}$ to zero. We find:

$$H_Y^* = \frac{\rho+\frac{1-\xi+\sigma(\xi-\gamma)}{1-\gamma}\delta H}{\left[\frac{(1-\gamma/\xi)\gamma}{n\alpha}+\frac{1-\xi+\sigma(\xi-\gamma)}{1-\gamma}\right]\delta} \qquad (17)$$

$$z^* = \frac{\rho+\sigma\frac{(\xi-\gamma)}{1-\gamma}\delta(H-H_Y^*)}{(1-(1-\gamma/\xi)/n)\,\gamma H_Y^{*\alpha}L^{\beta}} \qquad (18)$$

$$q^* = H_Y^{*\alpha}L^{\beta}z^*-\frac{(\xi-\gamma)}{1-\gamma}\delta(H-H_Y^*). \qquad (19)$$

The above formulae are only valid when H_Y^* is an interior solution. Since the technology in the R&D sector is linear in H_Y, a corner solution such as $H_Y^*=H$ may arise. For example, when n is too large, the profit generated by a patent is so tiny that no R&D is done. In our following discussion on the comparative dynamics, we implicitly assume

$$\rho < \frac{(1-\gamma/\xi)\,\gamma}{n\alpha}\delta H$$

to ensure an interior solution.

To avoid repeating what is discussed in Romer (1990), I will focus on the effect of n and ξ on the long-term rate of growth of output, which can be expressed as

$$g_Y = g_C = g_K = \left[\frac{\xi-\gamma}{1-\gamma}\right]\delta(H - H_Y^*). \tag{20}$$

An increase in n reduces the value of a patent and hence reduces H_A^*. Thus H_Y^* increases with n, which is confirmed by our formula (17). The rate of growth of output is thus lower. To see the effect of ξ on g_Y, let us rewrite the expression (20) as,

$$g_Y = \frac{\left[\frac{(1-\gamma/\xi)\gamma}{n\alpha}\delta H - \rho\right]}{\left[\frac{(1-\gamma)\gamma}{n\alpha\xi} + \frac{1-\xi}{(\xi-\gamma)} + \sigma\right]}$$

which states that as ξ increases, the denominator gets smaller whereas the numerator gets bigger, hence g_Y increases. This conforms with our intuition that as different types of goods become less substitutable, the value of R&D activity becomes more valuable and the long-term economic growth is greater.

The real interest rate on the balanced growth path can be calculated as follows:

$$r^* = (1 - (1-\gamma/\xi)/n)\,\gamma H_Y^{*\alpha} L^\beta z^*. \tag{21}$$

Suppose, for simplicity, that $A(0) = 1$. If the economy starts with $z(0) =$

z^*, we can compute the welfare easily, because, in this case,

$$\begin{aligned} C(0) &= q^* K(0) \\ &= q^* \left[z^*/A(0)^{\xi-\gamma} \right]^{1/(\gamma-1)} \\ &= q^* \left[z^* \right]^{1/(\gamma-1)}. \end{aligned}$$

Thus, the welfare can be directly computed as

$$V^*(n) = \frac{1}{1-\sigma} \left[\frac{\left(q^* \left[z^* \right]^{1/(\gamma-1)} \right)^{1-\sigma}}{\rho - (1-\sigma)(r^* - \rho)/\sigma} - \frac{1}{\rho} \right], \tag{22}$$

which is used in the comparative dynamics analysis below.

3 Comparative Dynamics Analysis

The parameter values I choose for the model are as follows: $\alpha = 1/3$, $\beta = 1/3$, $\gamma = 1/3$, $\xi = 1$, $\delta = 0.001$, $H = 100$, $L = 1$, $\sigma = 0.8$, $\rho = 0.01$, $A(0) = 1$. $K(0)$ is chosen such that $z(0) = z^* \mid_{n=5}$. I would like to find out how the welfare depends on n. When n is too large, we will encounter corner solutions that require delicate treatment in the welfare calculation. Thus we restrict ourself to $n = 1, 2, ..., 5$.

Given that we let $z(0) = z^* \mid_{n=5}$, the welfare for $n = 5$ can be calculated directly from (22). $V(n)$ for $n = 1, 2, 3, 4$ are calculated using the "reverse shooting" method. This choice of $z(0)$ is to make sure that a corner solution does not arise in calculating $V(n)$ for $n = 1, 2, ...5$.

Table 1 shows that as we break up the monopoly to 2, 3, 4, or 5 independent companies through regulations, the welfare drops by 63, 73, 77, and 78 percent. The drop in welfare is very substantial.

Table 1: $\sigma = 0.8,\ \xi = 1,\ H = 100$

n	1	2	3	4	5
welfare	100	37	27	23	22
long-term growth rate	3.86	2.06	1.20	0.69	0.06

If I change the value of ξ to $\xi = 0.8$, in other words, when the different types of equipment are substitutes, how much will the drop in welfare as the result of the break up be moderated?

Table 2 gives the answer. The welfare drops by 33, 45, 48, and 51 percent, respectively. The drop is still substantial, but quite dramatically moderated compared with the case of $\xi = 1$. Of course, as ξ approaches γ, the R&D sector becomes less important because the incentive for R&D is low in any case, and the drop in welfare will become insignificant.

Table 2: $\sigma = 0.8,\ \xi = 0.8,\ H = 100$

n	1	2	3	4	5
welfare	100	67	55	52	49
long-term growth rate	2.34	1.16	0.62	0.31	0.12

The discussion above seems to hint that as long as different types of equipment are not close substitutes, the benefit from increased efficiency in the manufacturing sector can hardly compensate for the harm done to long-term growth.

It turns out the this conclusion is pre-mature. The above conclusion largely depends on the low value of σ, i.e. the high intertemporal rate of substitution. Empirical studies on equity premium indicate that a large σ (low intertemporal rate of substitution) is more likely the case. Once I raise the value of σ to 2, the welfare drop is almost zero as Table 3 and 4 illustrate.

Table 3 shows the case of $\sigma = 2$ and $\xi = 1$. The welfare drops by 0.3 percent only when a monopoly is broken into 2 independent firms. Even when it is broken into 5 firms, the loss is only 1.2 percent. Table 4 shows the case of $\sigma = 2$ and $\xi = 0.8$. Here, the loss is only 1.0 percent when a monopoly is broken up into 5 firms.

Table 3: $\sigma = 2,\ \xi = 1,\ H = 100$

n	1	2	3	4	5
welfare	100	99.7	99.4	99.0	98.8
long-term growth rate	2.12	1.00	055	0.31	0.16

Table 4: $\sigma = 2,\ \xi = 0.8,\ H = 100$

n	1	2	3	4	5
welfare	100	99.7	99.4	99.2	99.0
long-term growth rate	1.48	0.67	0.35	0.17	0.06

Let us see what the intuition is that a large σ tends to raise the efficiency gains in the manufacturing sector from "breaking up" and to lower the damage on the long-run growth. A large σ means a low elasticity of intertemporal rate of substitution. Other things being equal, an agent would prefer to have a more smoothed consumption pattern instead of a steeply rising one. "Breaking up", indeed, acts to raise the current production and reduce the long-term rate of growth. Although the efficiency gains are still not enough to compensate for the loss of growth, the welfare cost is insignificant.

Table 5 puts the comparison in a clearer perspective. In Table 5, I raise the total stock of human capital to $H = 500$. This magnifies the growth differences. Indeed, in the case of $n = 1$, the long-term growth rate of output is 8.64 percent, much higher than the corresponding rate (1.86 percent) in the case of $n = 5$. This huge loss in long-term growth is however almost fully compensated by the efficiency gains in the manufacturing sector; the welfare when $n = 5$ is only 0.5 percent lower, compared with the welfare in the case of $n = 1$.

Table 5: $\sigma = 2,\ \xi = 0.8,\ H = 500$

n	1	2	3	4	5
welfare	100	99.9	99.7	99.6	99.5
long-term growth rate	8.64	4.77	3.22	2.39	1.86

This gives me the reason to speculate that the tiny disadvantage in "breaking up" in the case of a realistically low intertemporal rate of substitution may be overcome in Jones' R&D-based growth model (1995a). Jones (1995b) uses time series evidence to discredit the linearity assumption in the R&D technology in Romer (1990). He advocates that the technology should be modified as in Jones (1995a), namely,

$$\dot{A} = \delta H_A A^{\phi},$$

where he imposes the condition that $0 < \phi < 1$. An immediate conclusion of this type of technology is that the long-term rate of growth is independent of H_A and in fact it only depends on the rate of population growth and the parameter value ϕ. Hence, "breaking up" will have no impact on the long-term rate of growth but it will still keep the efficiency gains intact. Intuitively, there should exist an optimal degree of "breaking up", $n > 1$, in such a model[2].

[2]Again, for readers who are better equipped with computation tools than I am, the details of the differential equations are given in the appendix.

4 Conclusion

In Romer's endogenous growth model (1990), whether there is a serious welfare loss from a regulation that breaks up a monopoly into independent companies depends critically on the intertemporal rate of substitution. With a low intertemporal rate of substitution, the representative agent dislikes consumption variability across time, and the harm done to long-term growth is almost fully compensated by the benefit received in the manufacturing sector. Table 5 is particularly telling.

From this exercise, I am confident that in a growth model such as Jones (1995a), a regulation that breaks up every monopoly might even enhance the welfare if the intertemporal rate of substitution is low and the substitutability of different types of machines is sufficiently high. The reason is that in Jones' model, a break-up will have no impact on long-run growth, which is determined by the population growth rate and a fixed parameter in the R&D sector.

Indeed, 'To Break up or Not To Break up' depends on which model we believe in, the endogenous growth model or a semi-endogenous one. It is important to identify a model that better describes the real world because policy recommendations drawn in an endogenous growth model and those drawn in a semi-endogenous one are likely to be just the opposite.

If we interpret our analysis as one for a particular industry, the conclusion is as follows. The patent for specialized equipment in this industry should be licensed to a larger number of independent suppliers if, the intertemporal rate of substitution for the final consumption good in the industry is lower; the substitutability of specialized equipment is higher; and the parameter ϕ is lower.

Appendix: Transitional Dynamics in Jones' R&D-based Model (1995)

Jones (1995b) criticizes the scale effect presented in R&D-based Romer-type models. He reports times series evidence, showing that the long-term implications of these models are strongly rejected by data. He proposes to replace the technology of the R&D sector by

$$\dot{A} = \delta H_A A^{\phi},$$

where he imposes the condition that $0 < \phi < 1$.

If human capital stock were fixed as in Romer (1990), then the above change of R&D technology would make long-term growth impossible due to the restriction that $\phi < 1$. To allow for long-run growth, Jones assumes that human capital stock grows at a constant rate

$$\dot{H} = gH.$$

It is then easy to see that in the long-run,

$$\frac{\dot{A}}{A} = \frac{g}{1-\phi},$$

which will also determine the rate of growth of capital and output.

The rest of the model is the same as in our extended Romer model. Before I go into the details of the analysis, it is worthwhile to work on the intuition. Since a forced break up, albeit reducing the value of a patent, has no impact on the long-term rate of growth, such a policy is more likely to raise the welfare as a result of improved efficiency in the manufacturing sector.

Again, following Mulligan and Sala-i-Martin (1991), I redefine our variables as follows:

$$h_Y = H_Y/H$$

$$z = A^{\xi-\gamma+\alpha(1-\phi)} K^{\gamma-1}$$
$$q = \frac{C}{K}$$

and

$$a = A^{1-\phi}/H.$$

We know that these variables will all converge to their steady-state values in the long run.

The equilibrium path is characterized by following differential equations in a, z, h_Y and q. The first two variables are state-like and the latter two are control-like.

$$\frac{\dot{a}}{a} = (1-\phi)\delta\frac{(1-h_Y)}{a} - g$$

$$\frac{\dot{z}}{z} = (\xi - \gamma + \alpha(1-\phi))\,\delta\frac{(1-h_Y)}{a} + (\gamma - 1)\left(\left(\frac{h_Y}{a}\right)^{\alpha} z - q\right)$$

$$\begin{aligned}
\frac{\dot{h}_Y}{h_Y} &= \frac{1}{1-\alpha}\left[\begin{array}{c} \gamma\left(\left(\frac{h_Y}{a}\right)^{\alpha} z - q\right) + (\xi-\phi-\gamma)\delta\frac{(1-h_Y)}{a} - (1-\alpha)g \\ +\delta\frac{[(1-\gamma/\xi)/n]\gamma}{\alpha}\left(\frac{h_Y}{a}\right) - [1-(1-\gamma/\xi)/n]\,\gamma\left(\frac{h_Y}{a}\right)^{\alpha} z \end{array}\right] \\
&= \frac{1}{1-\alpha}\left[(\xi-\phi-\gamma)\delta\frac{(1-h_Y)}{a} + \delta\frac{[(1-\gamma/\xi)]\,\gamma}{n\alpha}\left(\frac{h_Y}{a}\right) + \frac{(1-\gamma/\xi)}{n}\gamma\left(\frac{h_Y}{a}\right)^{\alpha} z - \gamma q\right]
\end{aligned}$$

$$\begin{aligned}
\frac{\dot{q}}{q} &= \frac{\gamma\left(\frac{h_Y}{a}\right)^{\alpha} z\,[1-(1-\gamma/\xi)/n] - \rho}{\sigma} - \left(\left(\frac{h_Y}{a}\right)^{\alpha} z - q\right) \\
&= \frac{[[1-(1-\gamma/\xi)/n]\,\gamma - \sigma]\left(\frac{h_Y}{a}\right)^{\alpha} z}{\sigma} + q - \frac{\rho}{\sigma}.
\end{aligned}$$

Let us now calculate the steady-state values of these variables. From $\dot{a}/a = 0$, we find

$$\delta\frac{(1-h_Y^*)}{a^*} = \frac{g}{1-\phi}. \tag{23}$$

From $\dot{z}/z = 0$ and $\dot{q}/q = 0$, we obtain

$$(\xi - \gamma + \alpha(1-\phi))\frac{g}{1-\phi} = (1-\gamma)\left(\left(\frac{h_Y^*}{a^*}\right)^\alpha z^* - q^*\right)$$

$$r^* = \rho + \sigma\frac{(\xi - \gamma + \alpha(1-\phi))}{1-\gamma}\frac{g}{1-\phi}$$

$$\left(\frac{h_Y^*}{a^*}\right)^\alpha z^* = q^* + \frac{(\xi - \gamma + \alpha(1-\phi))}{1-\gamma}\frac{g}{1-\phi}. \tag{24}$$

From $\dot{h}_Y/h_Y = 0$, we know that

$$\frac{\delta\gamma\left[(1-\gamma/\xi)\right]}{n\alpha}\frac{h_Y^*}{a^*} + \frac{(1-\gamma/\xi)}{n}\gamma\left[q^* + \frac{(\xi - \gamma + \alpha(1-\phi))}{1-\gamma}\frac{g}{1-\phi}\right]$$
$$= \gamma q^* - \left[\xi - \gamma + \alpha(1-\phi) - 1\right]\frac{g}{1-\phi}.$$

Since

$$r^* = \gamma\left[1 - (1-\gamma/\xi)/n\right]\left(\frac{h_Y^*}{a^*}\right)^\alpha z^*$$

can be used to solve for

$$\left(\frac{h_Y^*}{a^*}\right)^\alpha z^* = \frac{1}{\left[1 - (1-\gamma/\xi)/n\right]\gamma}r^*,$$

substitute this back into equation (24), and we can compute

$$q^* = \frac{1}{\left[1 - (1-\gamma/\xi)/n\right]\gamma}r^* - \frac{(\xi - \gamma + \alpha(1-\phi))}{1-\gamma}\frac{g}{1-\phi}.$$

Then from equation (25), we solve for

$$\frac{h_Y^*}{a^*} = \frac{\gamma q^* - \frac{(1-\gamma/\xi)}{n}\frac{1}{[1-(1-\gamma/\xi)/n]\gamma}r^* - \left[\xi - \gamma + \alpha(1-\phi) - 1\right]\frac{g}{1-\phi}}{\frac{\delta\gamma[(1-\gamma/\xi)]}{n\alpha}}, \tag{26}$$

which can then be used to solve for

$$z^* = \frac{\frac{1}{[1-(1-\gamma/\xi)/n]\gamma}r^*}{\left(\frac{h_Y^*}{a^*}\right)^\alpha}. \tag{27}$$

Finally, a^* and h_Y^* can be solved from (23) and (26).

References

[1] Barro R. 1990. "Government Spending in a Simple Model of Endogenous Growth," *Journal of Political Economy*, Vol. 98. S103–S125.

[2] Benhabib J., R. Perli and D. Xie, 1994. "Monopolistic Competition, Indeterminacy and Growth," *Ricerche Economiche*, Vol 48, 279–298.

[3] Davidson C. and P. Segerstrom, 1992. "Patent Enforcement and Economic Growth, " Mimeo, Michigan State University.

[4] Deverajan S., D. Xie and H. Zou, 1998. "Should Public Capital Be Subsidized or Provided," *Journal of Monetary Economics*, Vol. 41, 319–331.

[5] Futagami K., K. Mino., and Y. Ohkusa, 1999. "Patent Length and Economic Growth," Mimeo, Osaka University.

[6] Glomm, G., and B. Ravikumar, 1994. "Public Investment in Infrastructure in a Simple Growth Model," *Journal of Economic Dynamics and Control*, Vol. 18, 1173–1187.

[7] Horowitz, A. and E. Lai, 1996. "Patent Length and The Rate of Innovation," *International Economic Review*, Vol. 37, 785–801.

[8] Jones, C. I., 1995a. "R&D-Based Models of Economic Growth," *Journal of Political Economy*, Vol. 103: 759–784.

[9] Jones, C. I., 1995b. "Time Series Tests of Endogenous Growth Models," *Quarterly Journal of Economics*, Vol. 110, 495–525.

[10] Judd K. 1998. Numerical Methods in Economics, The MIT Press.

[11] Kremer M. 1998. "Patent Buyouts: A Mechanism for Encouraging Innovation," *Quarterly Journal of Economics*, Vol. 113, 1137–1167.

[12] Mino K., 1996. "Analysis of a two-sector model of endogenous growth with capital income taxation" *International Economic Review*, Vol. 37, 227-251.

[13] Mulligan, C. B. and X. Sala-i-Martin, 1991. "A Note on the Time-Elimination Method for Solving Recursive Economic Models," NBER Technical Working Paper, No. 116.

[14] Romer P. M., 1990. "Endogenous Technological Change," *Journal of Political Economy*, Vol. 98, S71–S102.

[15] Rebelo S., 1991. "Long-Run Policy Analysis and Long-Run Growth," *Journal of Political Economy*, Vol. 99, 500–521.

[16] Stokey N. and S. Rebelo, 1995. "Growth Effects of Flat-Rate Taxes," *Journal of Political Economy*, Vol. 103, 519–550.